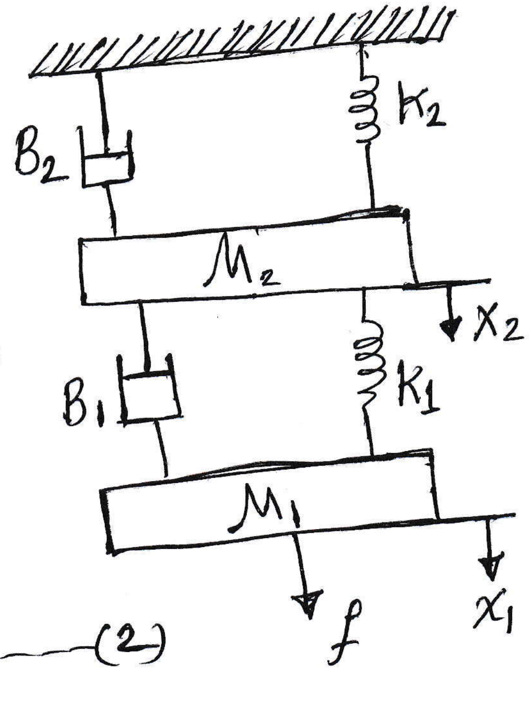
	Electrical system	Translation system	Rotational system
1-	Current i	Force f	Torque T
2-	Voltage v	Velocity u	angular velocity ω
3-	Flux linkages ψ	Displacement x	angular displacement θ
4-	Capacitor C	Mass M	Moment of Inertia J
5-	Conductance G	Damping coefficient B	Rotational Damping coefficient B
6-	Inductance L	Compliance $\frac{1}{K}$	Compliance $\frac{1}{K}$

EX Write the equations describing the motion of the mechanical system shown in figure below and find the transfer function $X_1(s)/F(s)$.

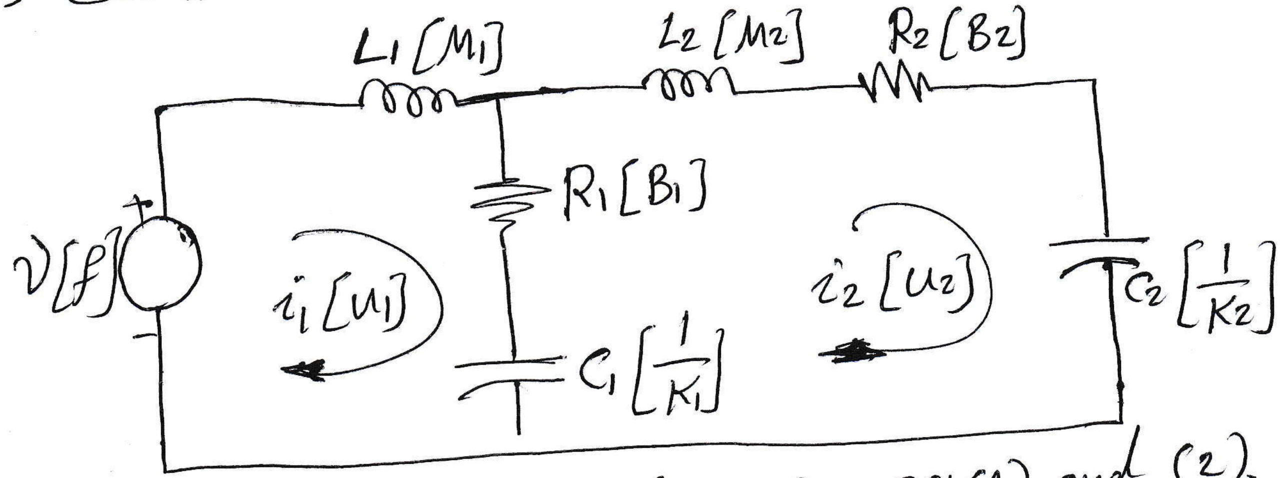
- force acting on mass M_1.



$$f = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{d(x_1 - x_2)}{dt} + K_1(x_1 - x_2) \quad \text{——(1)}$$

$$M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + B_1 \frac{d(x_2 - x_1)}{dt} + K_2 x_2 + K_1(x_2 - x_1) = 0 \quad \text{——(2)}$$

from eqns (1) & (2), force voltage analogous electrical circuits can be drawn as shown,



By taking Laplace Transform for eq (1) and (2).

$$F(s) = M_1 s^2 X_1(s) + B_1 s X_1(s) - B_1 s X_2(s) + K_1 X_1(s) - K_1 X_2(s) \quad \text{--- (3)}$$

$$M_2 s^2 X_2(s) + B_2 s X_2(s) + B_1 s X_2(s) - B_1 s X_1(s) + K_2 X_2(s) + K_1 X_2(s) - K_1 X_1(s) = 0 \quad \text{--- (4)}$$

- for eq (4)

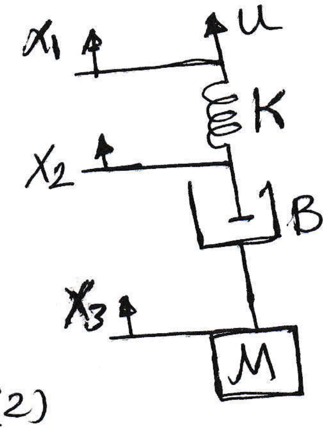
$$X_2(s) = \frac{B_1 s + K_1}{M_2 s^2 + (B_1 + B_2) s + (K_1 + K_2)} X_1(s)$$

Substituting $X_2(s)$ in eq (3)

$$X_1(s) = \frac{M_2 s^2 + (B_1 + B_2) s + K_1 + K_2}{(M_1 s^2 + B_1 s + K_1)\left[M_2 s^2 + (B_1 + B_2) s + (K_1 + K_2)\right] - (B_1 s + K_1)^2} F(s)$$

$$\therefore T(s) = \frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + (B_1 + B_2) s + K_1 + K_2}{(M_1 s^2 + B_1 s + K_1)\left[M_2 s^2 + (B_1 + B_2) s + (K_1 + K_2)\right] - (B_1 s + K_1)^2}$$

Ex obtain the f-v and f-i analogous circuits for the mechanical system shown in figure below and write down the equilibrium equations.



The equilibrium equations are:

$$K(x_1 - x_2) = f \qquad \text{—— (1)}$$

$$B(\dot{x}_2 - \dot{x}_3) + K(x_2 - x_1) = 0 \qquad \text{—— (2)}$$

$$B(\dot{x}_3 - \dot{x}_2) + M\ddot{x}_3 = 0 \qquad \text{—— (3)}$$

from eqs(1) and (2), we have

$$B(\dot{x}_2 - \dot{x}_3) = f \qquad \text{—— (4)}$$

from eqs (3) and (4), we have

$$M\ddot{x}_3 = f \qquad \text{—— (5)}$$

force Current Analogous Circuit

Replacing the electrical quantities in equation (1), (4), and (5) by their force-current analogous quantities, we have

$$\frac{1}{L}(\varphi_1 - \varphi_2) = i$$

or $$\frac{1}{L}\int(v_1 - v_2)\,dt = i \qquad \text{—— (6)}$$

$$G(\dot{\varphi}_2 - \dot{\varphi}_3) = i$$

or

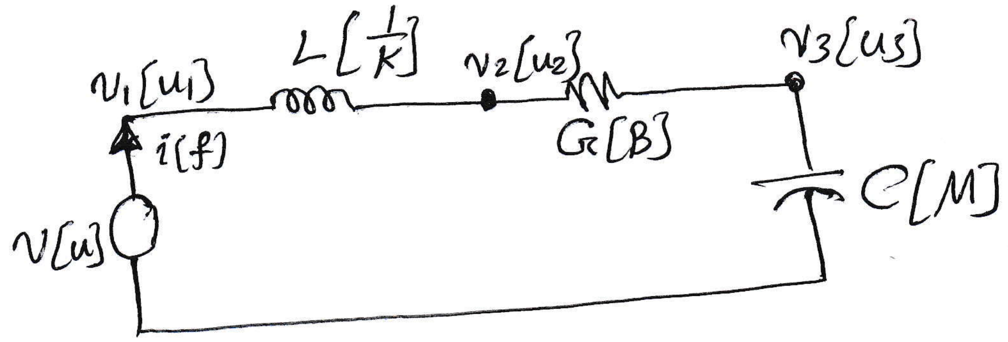
$$G(v_2 - v_3) = i \qquad \text{————} \quad (7)$$

$$C\ddot{\psi}_3 = i$$

or

$$C\frac{dv_3}{dt} = i \qquad \text{————} \quad (8)$$

if i is produced by a voltage source v, we have the electrical circuit based on $f-i$ analogy in figure below.



force voltage Analogous Circuit

Using force voltage analogy, the quantities in eq(1), (4), and (5) are replaced by the mechanical quantities to get,

$$\frac{1}{C}(q_1 - q_2) = v$$

or

$$\frac{1}{C}\int(i_1 - i_2)\,dt = v \qquad \text{————} \quad (9)$$

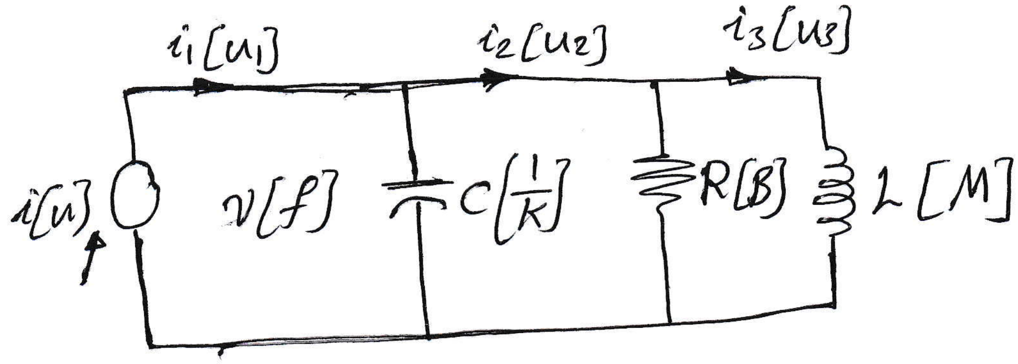
$$R(\dot{q}_2 - \dot{q}_3) = v$$

or

$$R(i_2 - i_3) = v$$

$$L\frac{d^2q_3}{dt^2} = v \quad \text{or} \quad L\frac{di_3}{dt} = v \qquad \text{——} \quad (10)$$

If the voltage is due to a current source i, we have the force voltage analogous circuits is shown in below.

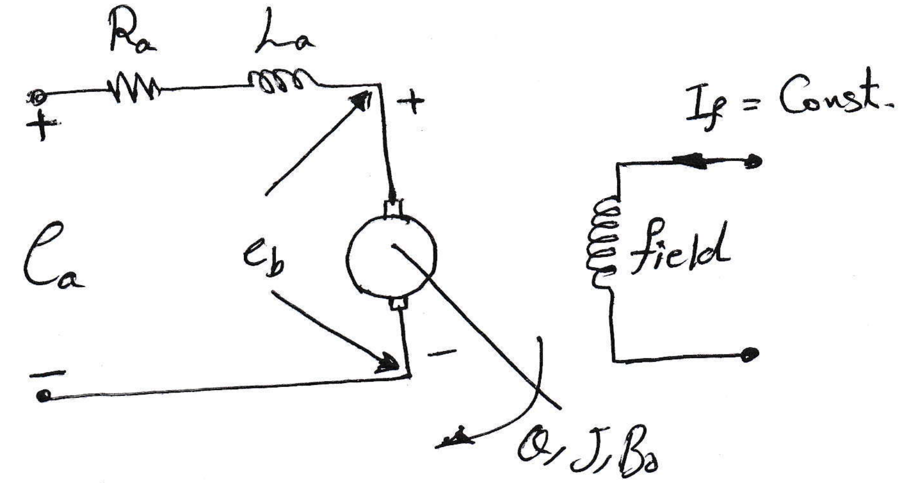


2-2-3 DC Servo Motor :-

A DC servo motor is used as an actuator to drive a load. It is usually a DC motor of low power ~~rating~~. DC servo motors have a high ratio of starting torque of inertia and therefore they have a faster dynamic response. DC motors are constructed using rare earth permanent magnet which have high residual flux density and high coercitivity.

In some application DC servo motors are used with magnetic flux produced by field windings. The speed of PMDC motors can be controlled by applying variable armature voltage. These are called armature voltage controlled DC servo motors. Wound field DC motors can be controlled by either controlling the armature voltage or Controlling the field current.

(a) Armature controlled DC servo motor :-



the Torque produced by the motor is given by,

$$T = K_T \, i_a$$

K_T : is the motor torque Constant.

the back emf is proportional to the speed of the motor and hence

$$e_b = K_b \dot{\theta}$$

The differential equation representing the electrical System is given by,

$$R_a i_a + L_a \frac{di_a}{dt} + e_b = e_a$$

By taking Laplace transform for all equations.

$$T(s) = K_T\, I_a(s) \qquad \text{——— (1)}$$

$$E_b(s) = K_b\, s\, \theta(s)$$

$$(R_a + sL_a)\, I_a(s) + E_b(s) = E_a(s)$$

$$I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + sL_a} \qquad \text{——— (2)}$$

The mathematical model of the mechanical system is given by,

$$J \frac{d^2\theta}{dt^2} + B_0 \frac{d\theta}{dt} = T$$

Taking Laplace transform

$$J s^2 \theta(s) + B_0 s\, \theta(s) = T(s) \qquad \text{——— (3)}$$

using eqns (1) and (2) in eq (3), we have

$$\theta(s) = K_T \frac{E_a(s) - K_b s\, \theta(s)}{(R_a + sL_a)(Js^2 + B_0 s)}$$

Solving for $\theta(s)$, we get

$$\theta(s) = \frac{K_T\, E_a(s)}{s\left[(R_a + sL_a)(Js + B_0) + K_T K_b\right]} \qquad \text{--- (4)}$$

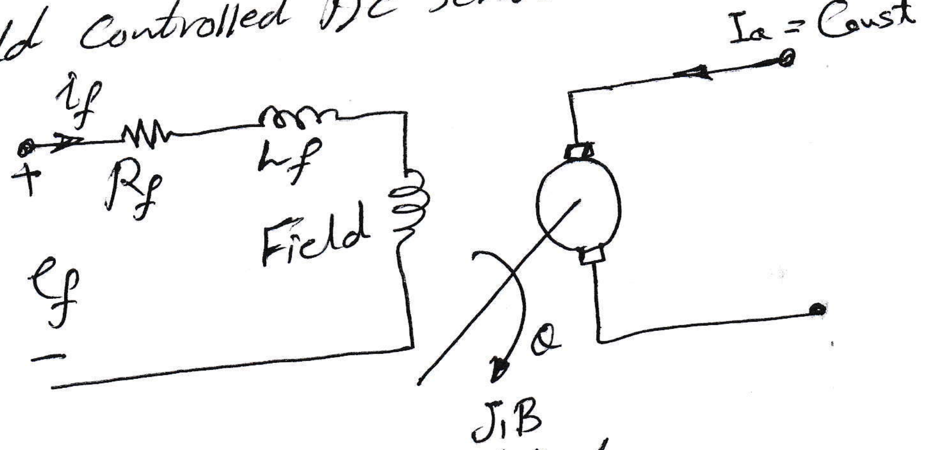
Now, the transfer function between the input $E_s(s)$ and the output $\theta(s)$, we get from eq (4)

$$T(s) = \frac{\theta(s)}{E_a(s)} = \frac{K_T/R_a}{s\left[Js + B_o + \frac{K_b K_T}{R_s}\right]}$$

$$= \frac{K_T/R_a}{s[Js+B)}, \quad B = \underbrace{B_o + \frac{K_b K_T}{R_a}}_{\text{equivalent frictional coefficient.}}$$

(b) Field Controlled DC servo motor

The field controlled DC servo moto is shown as below



The electric circuit is modelled as,

$$I_f(s) = \frac{E_f(s)}{R_f + L_f S} \quad \text{—— (1)}$$

$$T(s) = K_T \, I_f(s) \quad \text{—— (2)}$$

$$(Js^2 + B_o)\,\theta(s) = T(s) \quad \text{—— (3)}$$

Combining eq(1), eq(2) and (3), we have

$$\frac{\theta(s)}{E_f(s)} = \frac{K_T}{s(Js+B_o)(R_f + L_f s)}$$

$$= \frac{K_T / R_f B_0}{s\left[\frac{J}{B_0} s+1\right)\left(\frac{L_f}{R_f} s+1\right)}$$

$$= \frac{K_m}{s(\tau_m s+1)(\tau_f s+1)}$$

$$K_m = \frac{K}{R_f B_0} = \text{motor gain Constant.}$$

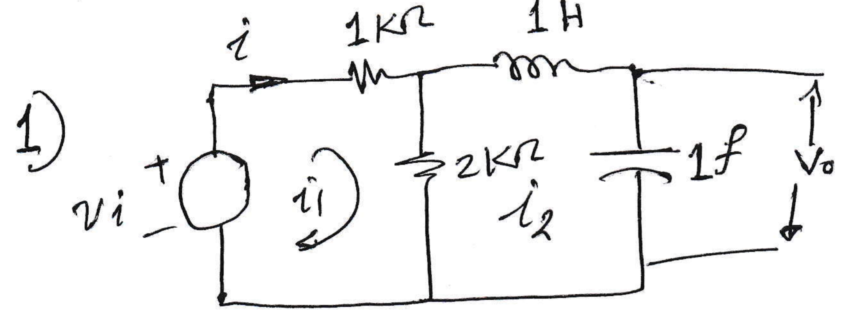
$$\tau_m = \frac{J}{B_0} = \text{motor time Constant.}$$

$$\tau_f = \frac{L_f}{R_f} = \text{field time Constant.}$$

Modeling and T.F.   Home Work 1   Control

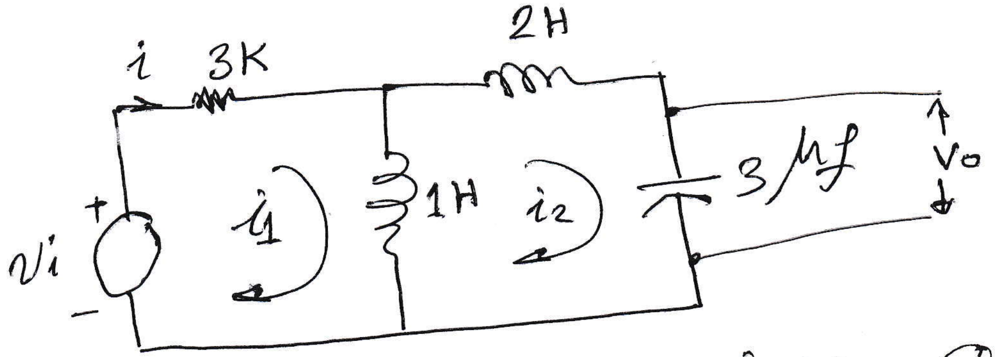
Q1
Find the transfer function of these figure.

1)



ⓐ $\frac{V_o(s)}{V_i(s)}$, ⓑ $\frac{V_o(s)}{i(s)}$, ⓒ $\frac{i_2(s)}{i(s)}$, ⓓ $\frac{i(s)}{V_i(s)}$

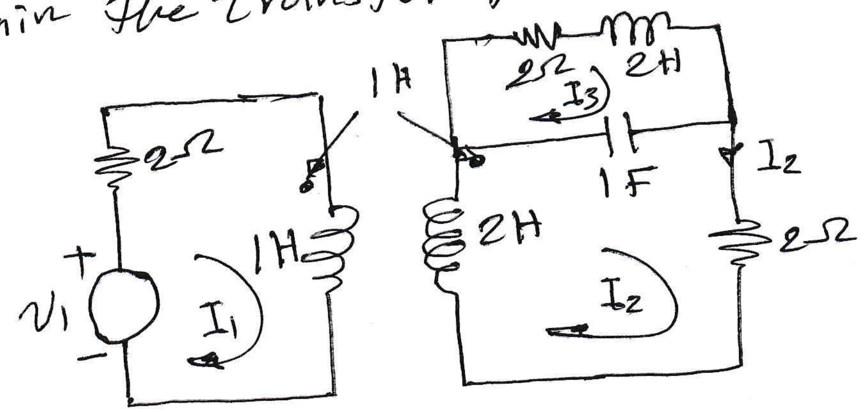
②



ⓐ $\frac{V_o(s)}{V_i(s)}$, ⓑ $\frac{V_o}{i(s)}$, ⓒ $\frac{i_2(s)}{i(s)}$, ⓓ $\frac{i(s)}{V_i(s)}$

Q2 obtain the transfer function for the network

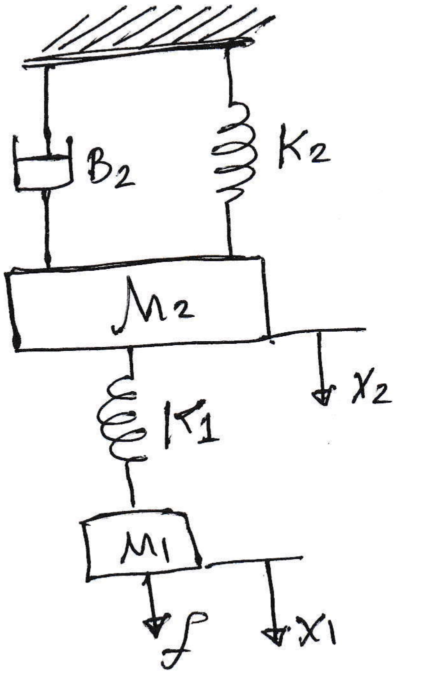
$$T(s) = \frac{I_2(s)}{V_1(s)}$$



Q3 obtain the transfer function for the following mechanical translational systems.

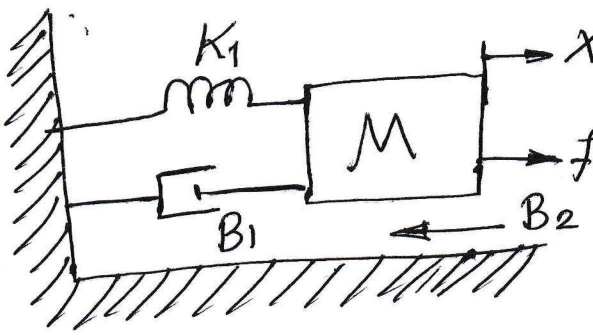
ⓐ

$$T(s) = \frac{X_2(s)}{F(s)}$$

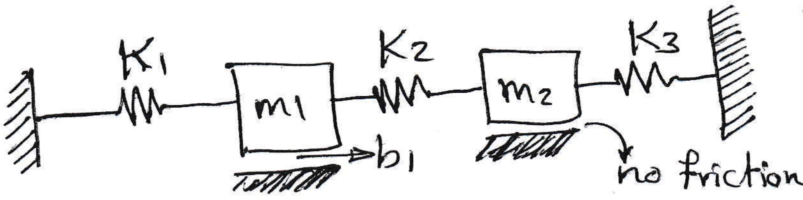


b)

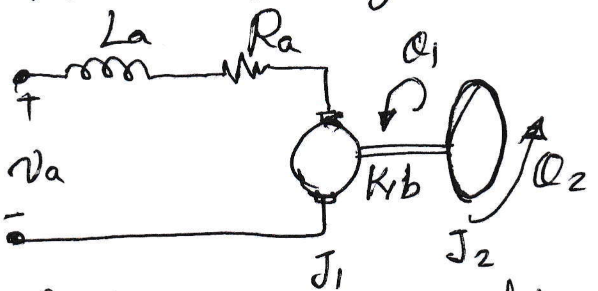
$$T(s) = \frac{X(s)}{F(s)}$$



ⓒ



Q4 write the equations of motion of figure below.



K_e : electric constant.
K_t : torque constant.
L_a : armature inductance
R_a : Resistance
J_1 : inertia of rotor ; B : viscous friction ; J_2 : inertia of Load.
K : spring constant   b : viscous damping.